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Reliability of the Estimation of CP Asymmetries for Nonleptonic $B^0 - \bar{B}^0$ Decays into Non-CP-Eigen states

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Abstract

CP asymmetries for two-body-nonleptonic $B^0 - \bar{B}^0$ decay into non-CP-eigen states are calculated using two different methods: (i) Bauer, Stech, and Wirbel factorization method to compute the decay amplitudes directly; (ii) using $B^0, \bar{B}^0$ decay amplitude ratios to avoid the direct computation of the decay amplitudes. The comparison of the results are made. The conclusion is presented.

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I. Introduction

The CP asymmetries for $B^0-\bar{B}^0$ two-body nonleptonic decays have been systematically estimated[1]. As proved in Ref.[1] (see its Appendix), for CP-eigen states f the amplitude ratios

$$\zeta = A(\bar{B}^0 \to f)/A(B^0 \to f)\ ,\ \ \bar{\zeta} = A(B^0 \to \bar{f})/A(\bar{B}^0 \to \bar{f})$$

depend only on KM matrix elements. But if the final state f is not a CP-eigen state, $\zeta,\ \bar{\zeta}$ are not pure KM factors. In the estimation of CP asymmetries in Ref.[1], the pure KM factor approximation for ζ and $\bar{\zeta}$ is also used for non-CP-eigen states, such as $D^{\pm}\pi^{\mp}$ etc. . But how reliable these estimations are? In this short article, we first compute the CP asymmetries for $B^0-\bar{B}^0$ decays into non-CP-eigen states using Bauer, Stech, Wirbel factorization method[2]. Then we compare these results with those by using the amplitude ratios ζ and $\bar{\zeta}$. In section II we present all the results of the two different methods. Section III devotes to the discussions and conclusions.

II. Computation of the partial-decay-rate asymmetries

For simplicity of comparison, we consider only incoherent $B_d^0-\bar{B}_d^0$ mesons. For instance, sitting on the Z^0 resonance, $b\bar{b}$ pairs will be produced in the form of $B_d^0B_u^-$ and $\bar{B}_d^0B_u^+$. Here $B_d^0,\ \bar{B}_d^0$ are produced incoherently and observing the charge of $B_u^-(B_u^+)$ would confirm the decayed neutral meson to be $B_d^0(\bar{B}_d^0)$. In the decays of incoherent $B_d^0-\bar{B}_d^0$ mesons, we can define the CP asymmetry parameter as

$$a_f(t) = \frac{\Gamma(B^0_{d,Phys.}(t) \to f) - \Gamma(B^0_{d,Phys.}(t) \to \bar{f})}{\Gamma(B^0_{d,Phys.}(t) \to f) + \Gamma(B^0_{d,Phys.}(t) \to \bar{f})} \qquad (2.1)$$

where

$$\begin{aligned} |B^0_{d,Phy.}(t)> &= f_+(t)|B_d^0> + \frac{q}{p}f_-(t)|\bar{B}_d^0> \\ |\bar{B}^0_{d,Phy.}(t)> &= \frac{p}{q}f_-(t)|B_d^0> + f_+(t)|\bar{B}_d^0> \end{aligned} \qquad (2.2)$$

$$\begin{aligned} |B_L> &= p|B_d^0> +q|\bar{B}_d^0> \\ |B_H> &= p|B_d^0> -q|\bar{B}_d^0> \end{aligned} \tag{2.3}$$

$$\begin{aligned} f_\pm(t) &= \frac{1}{2}(e^{-i\lambda_L t} \pm e^{-i\lambda_H t}) \\ \lambda_{L,H} &= m_{L,H} - \frac{i}{2}\Gamma_{L,H} \end{aligned} \tag{2.4}$$

Integrating with time t from zero to infinity we can get the integrated CP asymmetry

$$\mathcal{A}_f = \int_0^\infty dt \quad a_f(t) \tag{2.5}$$

Now we first compute the asymmetry parameter $\mathcal{A}_f$ in Bauer, Stech, Wirbel (BSW) scheme.

We take $B_d^0 \to D^+\pi^-$ as an example for the purpose of illustration.

For $B_d^0 \to D^-\pi^+$, the effective Hamiltonian is[2]

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud}\{a_1[\bar{b}\gamma_\mu(1-\gamma_5)c]_H[\bar{u}\gamma^\mu(1-\gamma_5)d]_H \\ &+ a_2[\bar{b}\gamma_\mu(1-\gamma_5)d]_H[\bar{u}\gamma^\mu(1-\gamma_5)c]_H\} + h.c. \end{aligned} \tag{2.6}$$

Neglecting the contribution of the exchange diagram we have

$$< D^-\pi^+|\mathcal{H}_{eff}(0)|B_d^- > \cong \frac{G_F}{\sqrt{2}} i f_\pi a_1 p_\pi^\mu < D^-|\bar{b}\gamma_\mu(1-\gamma_5)c|B_d^0 > \tag{2.7}$$

where

$$< \pi^+|\bar{u}\gamma^\mu(1-\gamma_5)d|0 >= i f_\pi p_\pi^\mu$$

has been used.

Using

$$\begin{aligned} < D^-|\bar{b}\gamma_\mu(1-\gamma_5)c|B_d^0 > &= [(p_B+p_D) - \frac{m_B^2-m_D^2}{q^2}q]_\mu F_1(q^2) \\ &+ \frac{m_B^2-m_D^2}{q^2} q_\mu F_0(q^2) \end{aligned} \tag{2.8}$$

and $q_\mu = (p_B - p_D)_\mu$, we finally get

$$< D^-\pi^+|\mathcal{H}_{eff}(0)|B_d^- > \approx \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} i f_\pi a_1 (m_B^2 - m_D^2) F_0^{BD}(m_\pi^2) \tag{2.9}$$

Similarly we can get

$$< D^-\pi^+|\mathcal{H}_{eff}(0)|\bar{B}_d^- >\approx \frac{G_F}{\sqrt{2}}V_{ub}V_{cd}^* i f_D a_1(m_B^2-m_\pi^2)F_0^{B\pi}(m_D^2) \tag{2.10}$$

Also

$$\begin{aligned} < D^+\pi^-|\mathcal{H}_{eff}(0)|B_d^- >\approx & \frac{G_F}{\sqrt{2}}V_{ub}^* V_{cd} i f_D a_1(m_B^2-m_\pi^2)F_0^{B\pi}(m_D^2) \\ < D^+\pi^-|\mathcal{H}_{eff}(0)|\bar{B}_d^- >\approx & \frac{G_F}{\sqrt{2}}V_{ud}^* V_{cb} i f_\pi a_1(m_B^2-m_D^2)F_0^{BD}(m_\pi^2) \end{aligned} \tag{2.11}$$

The CP asymmetry parameter

$$a_{D^-\pi^+}(t) = \frac{|< D^-\pi^+|\mathcal{H}_{eff}|B^0_{d,Phys.}(t) >|^2 - |< D^+\pi^-|\mathcal{H}_{eff}|\bar{B}^0_{d,Phys.}(t) >|^2}{|< D^-\pi^+|\mathcal{H}_{eff}|B^0_{d,Phys.}(t) >|^2 + |< D^+\pi^-|\mathcal{H}_{eff}|\bar{B}^0_{d,Phys.}(t) >|^2}$$

After the time integration we have

$$\mathcal{A}_{D^-\pi^+} = -\frac{0.514 f_\pi f_D m_B^2(m_B^2-m_D^2)x_d Im(\frac{V_{tb}^* V_{td} V_{cb} V_{ub} V_{ud}^* V_{cd}^*}{V_{tb}V_{td}^*})}{0.476 f_\pi^2(m_B^2-m_D^2)^2|V_{ud}^* V_{cb}|^2(2+x_d^2)+0.138 f_D^2 m_B^4 x_d^2|V_{cd}^* V_{ub}|^2} \tag{2.12}$$

where the values of the form factors are take from Ref.[2].

For the KM factors, we use the Wolfenstein parametrization[3]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2.13}$$

From the updated fit [4]

$$A\sim 0.84, \lambda\sim 0.22, \sqrt{\rho^2+\eta^2}\sim 0.36 \tag{2.14}$$

The values of ρ and η depend on the top quark mass m_t and $f_{B_d}\sqrt{B_{B_d}}$. For the purpose of illustration, we take $m_t\sim 174GeV$, $f_{B_d}\sqrt{B_{B_d}}\sim 180GeV$ and [4]

$$\rho = -0.05, \quad \eta = 0.33 \tag{2.15}$$

The other parameters are taken as

$$\begin{aligned} f_\pi &= 0.13GeV, \quad f_K = f_D = 0.16GeV \\ m_{B_d} &= 5.28GeV, \quad m_{D^+} = 1087GeV, \\ x_d &\sim 0.7 \end{aligned} \tag{2.16}$$

Substituting all these parameters into Eq.(2.12), we get the time-integrated CP asymmetry

$$\mathcal{A}_{D^-\pi^+} = -5.0 \times 10^{-3} \tag{2.17}$$

For the final states involved vector mesons, we use

$$< 0 \mid V_\mu \mid V >= \lambda_V m_V^2 \epsilon_\mu(V) \tag{2.18}$$

and take

$$\lambda_{D^*} = 0.14, \quad \lambda_\rho = 0.24 \tag{2.19}$$

Thus, we can use BSW method to compute the asymmetries for different processes. We put all these results in Table I. Note that for the final state f for which $B_d^0 \to f$ can occur but $\bar{B}_d^0 \to f$ cannot, there will be no CP asymmetry.

In order to compare these results with those by use of amplitude ratios, we compute the same CP asymmetries by use of the amplitude ratios like in Ref.[1] but use the same KM parameters as in BSW method. The results are presented also in Table I (denoted by AR method).

III. Discussions and conclusions

In Table I, we show both the results of BSW and AR (Amplitude Rations) methods. From that table we can see that for most of the processes listed there, the asymmetries by both methods are very close to each other. For very few processes, such as $f = D^-\rho^+$, there are large discrepancy but at most a factor 2 difference. So on the whole they agree to each other. At least the order of magnitudes of the CP asymmetries is reliable. But, if we want to use these CP asymmetries to make a precision test of the standard model, it is not good at all. For other purposes, the CP asymmetries computed by both BSW and AR still can be used. We must remind the reader that we are now talking about the non-CP-eigen states. If the final state is a CP-eigen state, the AR method can give a reliable prediction of the CP asymmetry.

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Table Captions

Table I. Asymmtretries in BSW and AR scheme. Here BSW means Bauer, Stech, Wirbel method [2], AR means Amplitude Ratio method [1].

Table I

Process	$\mathcal{A}_f(BSW)$	$\mathcal{A}_f(AR)$
$B_d^0 \to D^-\pi^+$	-5.0×10^{-3}	-6.6×10^{-3}
$D^+\pi^-$	-2.6×10^{-2}	-3.4×10^{-2}
$D^{*-}\pi^+$	9.1×10^{-3}	6.8×10^{-3}
$D^-\rho^+$	2.3×10^{-3}	6.8×10^{-3}
$\pi^-\rho^+$	0.24	0.37
$D^{*-}D^+$	-0.34	-0.25
D^-D^{*+}	-0.15	-0.26
π^-D^{*+}	4.6×10^{-2}	3.4×10^{-2}
$\pi^+\rho^-$	0.48	0.37
$\bar{D}^0\pi^0$	-7.0×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\eta$	-7.0×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\eta'$	-7.0×10^{-3}	-6.8×10^{-3}
$\bar{D}^{*0}\pi^0$	-6.9×10^{-3}	-6.8×10^{-3}
$\bar{D}^{*0}\eta$	-6.9×10^{-3}	-6.8×10^{-3}
$\bar{D}^{*0}\eta'$	-6.9×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\rho^0$	-7.1×10^{-3}	-6.8×10^{-3}
$\bar{D}^0\omega^0$	-7.1×10^{-3}	-6.8×10^{-3}
$D^0\pi^0$	-3.5×10^{-2}	-3.4×10^{-2}
$D^0\eta$	-3.5×10^{-2}	-3.4×10^{-2}
$D^0\eta'$	-3.5×10^{-2}	-3.4×10^{-2}
$D^{*0}\pi^0$	-3.5×10^{-2}	-3.4×10^{-2}
$D^{*0}\eta$	-3.5×10^{-2}	-3.4×10^{-2}
$D^{*0}\eta'$	-3.5×10^{-2}	-3.4×10^{-2}
$D^0\rho^0$	-3.6×10^{-2}	-3.4×10^{-2}
$D^0\omega^0$	-3.6×10^{-2}	-3.4×10^{-2}